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Tachyon matter

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Abstract: It is shown that classical decay of unstable D-branes in bosonic and superstring theories produces pressureless gas with non-zero energy density. The energy density is stored in the open string fields, even though around the minimum of the tachyon potential there are no open string degrees of freedom. We also give a description of this phenomenon in an effective field theory.

Keywords: Tachyon Condensation, D-branes, Conformal Field Models in String Theory.

Contents

1. Introduction and summary

In a previous paper [1] we developed a general formalism for constructing classical time dependent solutions describing the rolling tachyon field on unstable D-branes in bosonic and superstring theories. We also analysed the boundary state associated with these solutions in bosonic string theory in some detail, and showed that the energy momentum tensor approaches a finite limiting value if we push the system towards a direction in which the tachyon potential has a minimum.

In this paper we carry out a detailed study of the energy momentum tensor for rolling tachyon solution in D-branes in bosonic string theory, as well as for unstable D-branes and brane-antibrane configurations in superstring theories. We conclude that in both cases the asymptotic form of the energy momentum tensor for large time is described by a pressureless gas with non-zero energy density. The energy density is stored in the open string fields, even though around the minimum of the tachyon potential there are no open string degrees of freedom.

The paper is organised as follows. In section 2 we review the procedure used for determining the energy momentum tensor in terms of boundary states following [2, 3, 4], and determine the expression for the energy momentum tensor for the rolling tachyon solution in bosonic string theory. In sections 3 and 4 we generalize this procedure to unstable D-branes in superstring theories. In both theories the energy density remains constant and the pressure approaches zero as the tachyon field rolls towards its minimum. In section 5 we give a description of this phenomenon using an effective field theory.

It will be interesting to see if this kind of tachyon matter system can play some role in cosmology; in particular if it could contribute to the dark matter in the universe. Also interesting is the question as to whether tachyon condensation in other kind of unstable brane systems (e.g. branes at angles) could give rise to other kind of matter, with different equation of state. Another relevant issue is the nature of supersymmetry breaking induced by the tachyon matter. Since the total energy of the tachyon matter is an adjustable parameter, determined by the initial position and velocity of the tachyon, the associated supersymmetry breaking scale will also be an adjustable parameter.

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Before we conclude this section we should point out that the classical solutions described in this paper correspond to a special choice of initial conditions in which the the tachyon field is taken to be spatially homogeneous and no other field is excited. We could consider relaxing this restriction by choosing spatially inhomogeneous tachyon. This will correspond to perturbing the original boundary CFT by terms involving the coordinates tangential to the D-brane. In this case we should expect part of the initial energy of the system to go into creation of lower dimensional brane - antibrane pairs, and in general the system should decay into a mixture of tachyon matter described here and lower dimensional branes. However as long as the perturbation does not involve the transverse coordinates, the energy density is still firmly localized on the plane of the initial brane. If however we let the boundary perturbation depend on the transverse coordinates, then the energy density can leave the plane of the brane, and a general final state is expected to contain a mixture of tachyon matter and (excited) lower dimensional branes moving in directions transverse as well as tangential to the original brane. Finally, once quantum effects are taken into account, the results will be further modified, but we would expect that the time scale over which the quantum effects set in will go to infinity in the limit in which the string coupling goes to zero.

2. Analysis in bosonic string theory

A boundary state $|\mathcal{B}\rangle$ associated with a D-brane system in bosonic string theory is a closed string state of ghost number 3, defined as follows. For a closed string state $|\psi_c\rangle$ of ghost number 3 (which could be obtained for example by applying the operator $(c_0 - \bar{c}_0)$ on a physical closed string state of ghost number 2) $\langle\mathcal{B}|\psi_c\rangle$ gives the one point function of the closed string vertex operator ψ_c on the unit disk, with boundary condition appropriate to the particular D-brane system under consideration. The boundary state $|\mathcal{B}\rangle$ acts as a source for closed string fields; indeed it couples to the closed string field $|\Psi_c\rangle$ of ghost number 2, satisfying $(b_0 - \bar{b}_0)|\Psi_c\rangle = 0$ [5], through a term proportional to $\langle\mathcal{B}|(c_0 - \bar{c}_0)|\Psi_c\rangle$. To linearized order the equation of motion of $|\Psi_c\rangle$ in the presence of the D-brane system is given by:

$$(Q_B + \bar{Q}_B)|\Psi_c\rangle = |\mathcal{B}\rangle\,, \tag{2.1}$$

where Q_B and $\bar{Q}_B$ are the holomorphic and anti-holomorphic components of the BRST charge respectively. Since the closed string field includes the graviton modes, the boundary state $|\mathcal{B}\rangle$ contains information about the energy momentum tensor of the D-brane system. A simple way to determine the energy momentum tensor is to apply $(Q_B + \bar{Q}_B)$ on both sides of equation (2.1). This gives:

$$(Q_B + \bar{Q}_B)|\mathcal{B}\rangle = 0\,. \tag{2.2}$$

This equation contains information about the conservation laws of the closed string source, in particular of the energy momentum tensor. To see how this comes about, let us consider the level (1,1) states[1] in $|\mathcal{B}\rangle$ which are antisymmetric under the exchange of holomorphic

[1] We shall measure level by taking $(c_0 + \bar{c}_0)c_1\bar{c}_1|k\rangle$ to have level zero.

and anti-holomorphic modes of matter and ghost fields (since the graviton state is anti-symmetric under such an exchange). The general form of this part of the boundary state is given by:

$$|\mathcal{B}_0\rangle \propto \int d^{26}k \left[\widetilde{A}_{\mu\nu}(k)\alpha^{\mu}_{-1}\bar{\alpha}^{\nu}_{-1} + \widetilde{B}(k)(b_{-1}\bar{c}_{-1} + \bar{b}_{-1}c_{-1})\right](c_0+\bar{c}_0)c_1\bar{c}_1|k\rangle\,, \tag{2.3}$$

where $\widetilde{A}_{\mu\nu} = \widetilde{A}_{\nu\mu}$, and α^{μ}_n, b_n, c_n, $\bar{\alpha}^{\mu}_n$, $\bar{b}_n$, $\bar{c}_n$ are various matter and ghost oscillators. The anti-symmetry of the state under left-right exchange comes from the anti-symmetry of $c_1\bar{c}_1$ under such an exchange. Although we have included integration over all 26 components of the momentum, depending on the D-brane configuration, the function $\widetilde{A}_{\mu\nu}$ and $\widetilde{B}$ may have support only over a subspace of the 26-dimensional momentum space. In writing down (2.3) we have also used the condition that the boundary state is annihilated by $(b_0 - \bar{b}_0)$ and $(c_0 + \bar{c}_0)$.

In the $\alpha' = 1$ unit, we have

$$(Q_B+\bar{Q}_B)|\mathcal{B}_0\rangle \propto \sqrt{2}\int d^{26}k\left[k^{\nu}\widetilde{A}_{\mu\nu}(k) + k_{\mu}\widetilde{B}(k)\right](\bar{c}_{-1}\alpha^{\mu}_{-1} + c_{-1}\bar{\alpha}^{\mu}_{-1})(c_0+\bar{c}_0)c_1\bar{c}_1|k\rangle\,, \tag{2.4}$$

If $A_{\mu\nu}(x)$ and $B(x)$ denote the Fourier transforms of $\widetilde{A}_{\mu\nu}$ and $\widetilde{B}$ respectively, then the equation $(Q_B+\bar{Q}_B)|\mathcal{B}_0\rangle = 0$ gives us:

$$\partial^{\nu}(A_{\mu\nu}(x) + \eta_{\mu\nu}B(x)) = 0\,. \tag{2.5}$$

This shows that we should identify the conserved energy-momentum tensor as

$$T_{\mu\nu}(x) = K(A_{\mu\nu}(x) + \eta_{\mu\nu}B(x))\,, \tag{2.6}$$

where K is an appropriate normalization constant.

We shall now focus on the rolling tachyon solution of ref. [1]. For definiteness we focus on D-25-brane in flat space-time. For this system the boundary state $|\mathcal{B}\rangle$ is given by

$$|\mathcal{B}\rangle \propto |\mathcal{B}\rangle_{c=1}\otimes|\mathcal{B}\rangle_{c=25}\otimes|\mathcal{B}\rangle_{\text{ghost}}\,, \tag{2.7}$$

Here $|\mathcal{B}\rangle_{c=1}$ is the boundary state associated with the boundary CFT involving the X^0 field, $|\mathcal{B}\rangle_{c=25}$ is the boundary state associated with the $c = 25$ theory describing the flat space-like directions X^i along which we have Neumann boundary condition, and $|\mathcal{B}\rangle_{\text{ghost}}$ is the boundary state associated with the ghost CFT. $|\mathcal{B}\rangle_{c=25}$ and $|\mathcal{B}\rangle_{\text{ghost}}$ are given by, respectively,

$$|\mathcal{B}\rangle_{c=25} \propto \exp\left(-\sum_{n=1}^{\infty}\frac{1}{n}\alpha^i_{-n}\bar{\alpha}^i_{-n}\right)|0\rangle\,, \tag{2.8}$$

and

$$|\mathcal{B}\rangle_{\text{ghost}} \propto \exp\left(-\sum_{n=1}^{\infty}(\bar{b}_{-n}c_{-n} + b_{-n}\bar{c}_{-n})\right)(c_0+\bar{c}_0)c_1\bar{c}_1|0\rangle\,. \tag{2.9}$$

Finally as shown in [1], the relevant part of $|\mathcal{B}\rangle_{c=1}$ is given by:

$$|\mathcal{B}\rangle_{c=1} \propto \left[f(X^0(0)) + \alpha^0_{-1}\bar{\alpha}^0_{-1}g(X^0(0))\right]|0\rangle\,, \tag{2.10}$$

where

$$f(x^0) = \frac{1}{1 + e^{x^0}\sin(\widetilde{\lambda}\pi)} + \frac{1}{1 + e^{-x^0}\sin(\widetilde{\lambda}\pi)} - 1\,, \tag{2.11}$$

and,

$$g(x^0) = \cos(2\widetilde{\lambda}\pi) + 1 - f(x^0)\,. \tag{2.12}$$

$\widetilde{\lambda}$ is a parameter related to the initial displacement of the tachyon field. Putting these results together, collecting the level 1 states, and comparing the resulting expression with (2.3), we get

$$A_{00}(x) = g(x^0)\,, \qquad A_{ij}(x) = -f(x^0)\delta_{ij}\,, \qquad B(x) = -f(x^0)\,. \tag{2.13}$$

Hence, from (2.6), we have

$$T_{00} = K(f(x^0) + g(x^0)) = K(\cos(2\widetilde{\lambda}\pi) + 1)\,, \tag{2.14}$$

and

$$T_{0i} = 0\,, \qquad T_{ij} = -2Kf(x^0)\delta_{ij}\,. \tag{2.15}$$

The overall constant K can be fixed by requiring that at $\widetilde{\lambda} = 0$ the energy density must agree with the Dp-brane tension $\mathcal{T}_p$. This gives

$$K = \frac{1}{2}\mathcal{T}_p\,. \tag{2.16}$$

We see from (2.11) that for $0 < \widetilde{\lambda} \le \pi/2$, as $x^0 \to \infty$, $f(x^0) \to 0$. Thus $T_{ij} \to 0$ and the system has zero pressure. On the other hand the energy density T_{00} remains costant.

3. Generalization to superstrings

We now generalize these results to the case of unstable D-branes or brane-antibrane system in superstring theory. The boundary state for a D-brane in superstring theory is defined in the same way as in the case of bosonic string. For determining the energy-momentum tensor we can focus our attention on the NS-NS sector of the boundary state. The ghost number and picture number conservation laws on the disk and the sphere can be used to conclude that the NS-NS sector boundary state $|\mathcal{B}\rangle$ in superstring theory has the form:

$$|\mathcal{B}\rangle = \mathcal{O}|\Omega, k\rangle\,. \tag{3.1}$$

Here $\mathcal{O}$ is a ghost number zero operator constructed from the bosonic and fermionic matter oscillators α^μ_{-n}, $\bar{\alpha}^\mu_{-n}$, ψ^μ_{-n}, $\bar{\psi}^\mu_{-n}$, and oscillators of b, c, $\bar{b}$, $\bar{c}$ as well as the bosonic ghost fields β, γ, $\bar{\beta}$, $\bar{\gamma}$. $|\Omega, k\rangle$ is the ghost number 3, picture number -2 Fock vacuum with momentum k:

$$|\Omega, k\rangle = (c_0 + \bar{c}_0)c_1\bar{c}_1 e^{-\phi(0)} e^{-\bar{\phi}(0)} e^{ik.X(0)}|0\rangle\,, \tag{3.2}$$

where ϕ, $\bar{\phi}$ are bosonized ghost fields [6]. With such a $|\mathcal{B}\rangle$, $\langle\mathcal{B}|\psi_c\rangle$ will be non-zero for a ghost number 3, picture number -2 closed string vertex operator ψ_c, which are the correct quantum numbers for getting a non-zero one point function of ψ_c on the disk. Since $|\Omega\rangle$

is odd under both the left and the right moving GSO projection, and is symmetric under left-right exchange, the general level (1/2,1/2) GSO even contribution to the boundary state, anti-symmetric under left-right exchange, is given by:

$$|\mathcal{B}_0\rangle \propto \int d^{10}k \left(\widetilde{A}_{\mu\nu}(k)\psi^{\mu}_{-1/2}\bar{\psi}^{\nu}_{-1/2} + \widetilde{B}(k)\left(\bar{\beta}_{-1/2}\gamma_{-1/2} - \beta_{-1/2}\bar{\gamma}_{-1/2}\right)\right)|\Omega, k\rangle\,, \tag{3.3}$$

where $\widetilde{A}_{\mu\nu} = \widetilde{A}_{\nu\mu}$. Requiring $(Q_B + \bar{Q}_B)|\mathcal{B}_0\rangle = 0$ then gives:

$$k^{\mu}(\widetilde{A}_{\mu\nu} + \eta_{\mu\nu}\widetilde{B}) = 0\,. \tag{3.4}$$

As in the case of bosonic string theory, this lets us identify the energy momentum tensor $T_{\mu\nu}$ as:

$$T_{\mu\nu}(x) = K(A_{\mu\nu}(x) + B(x)\eta_{\mu\nu})\,, \tag{3.5}$$

for some constant K. Here $A_{\mu\nu}$ and B are Fourier transforms of $\widetilde{A}_{\mu\nu}$ and $\widetilde{B}$ respectively.

In section 4 we shall show that for the rolling tachyon solution on a D9 brane in type IIA or a D9-$\bar{\text{D}}$9 brane in type IIB superstring theory, we have:

$$A_{00}(x) = g(x^0)\,, \qquad A_{ij}(x) = -f(x^0)\delta_{ij}\,, \qquad B(x) = -f(x^0)\,, \tag{3.6}$$

where

$$f(x^0) = \frac{1}{1 + e^{\sqrt{2}x^0}\sin^2(\widetilde{\lambda}\pi)} + \frac{1}{1 + e^{-\sqrt{2}x^0}\sin^2(\widetilde{\lambda}\pi)} - 1\,, \tag{3.7}$$

$$g(x^0) = \cos(2\widetilde{\lambda}\pi) + 1 - f(x^0)\,. \tag{3.8}$$

$\widetilde{\lambda}$, as usual is the deformation parameter that labels the initial position of the tachyon field. Thus we have

$$T_{00} = K(g(x^0) + f(x^0)) = K(\cos(2\widetilde{\lambda}\pi) + 1)\,, \qquad T_{ij} = -2Kf(x^0)\delta_{ij}\,, \qquad T_{0i} = 0\,. \tag{3.9}$$

By requiring that at $\widetilde{\lambda} = 0$ T_{00} is equal to the total tension of the brane system, the constant K can be determined to be equal to half of the total tension of the brane system.

We see from (3.7) that for either sign of $\widetilde{\lambda}$, as $x^0 \to \infty$, $f(x^0) \to 0$. Thus we see again that in this limit T_{ij} and hence the pressure vanishes, whereas the energy density remains constant at $K(\cos(2\widetilde{\lambda}\pi) + 1)$.

4. Boundary state for rolling tachyon solution in superstrings

From general considerations, the NS-NS sector boundary state for the rolling tachyon solution in the case of superstring will be given by [7]–[10]:

$$|\mathcal{B}\rangle = |\mathcal{B}, +\rangle - |\mathcal{B}, -\rangle\,, \tag{4.1}$$

where

$$|\mathcal{B}, \epsilon\rangle \propto |\mathcal{B}, \epsilon\rangle_{X^0,\psi^0} \otimes |\mathcal{B}, \epsilon\rangle_{\vec{X},\vec{\psi}} \otimes |\mathcal{B}, \epsilon\rangle_{\text{ghost}}\,, \qquad \epsilon = \pm\,. \tag{4.2}$$

Here $(\vec{X},\vec{\psi})$ stand for the CFT of the spacelike coordinates $X^1,\dots X^9$ and their fermionic partners $\psi^1,\dots\psi^9,\bar\psi^1,\dots\bar\psi^9$. Of the different factors appearing in (4.2), $|\mathcal{B},\epsilon\rangle_{\vec{X},\vec{\psi}}$ and $|\mathcal{B},\epsilon\rangle_{\rm ghost}$ have the same form as on a static D9-brane:

$$|\mathcal{B},\epsilon\rangle_{\vec{X},\vec{\psi}} \propto \exp\left(-\sum_{n=1}^{\infty}\frac{1}{n}\alpha^j_{-n}\bar\alpha^j_{-n}\right)\exp\left(-i\epsilon\sum_{n=0}^{\infty}\psi^j_{-n-1/2}\bar\psi^j_{-n-1/2}\right)|0\rangle\,, \tag{4.3}$$

with sum over j running from 1 to 9, and

$$\begin{aligned}|\mathcal{B},\epsilon\rangle_{\rm ghost} \propto \exp&\left(-\sum_{n=1}^{\infty}\left(\bar b_{-n}c_{-n}+b_{-n}\bar c_{-n}\right)\right)\times\\ &\times\exp\left(-i\epsilon\sum_{n=1}^{\infty}\left(\bar\beta_{-n-1/2}\gamma_{-n-1/2}-\beta_{-n-1/2}\bar\gamma_{-n-1/2}\right)\right)|\Omega\rangle\,,\end{aligned} \tag{4.4}$$

where

$$|\Omega\rangle = (c_0+\bar c_0)c_1\bar c_1 e^{-\phi(0)}e^{-\bar\phi(0)}|0\rangle\,. \tag{4.5}$$

We now need to determine $|\mathcal{B},\epsilon\rangle_{X^0,\psi^0}$. We shall show that the part of $|\mathcal{B},\epsilon\rangle_{X^0,\psi^0}$ which involves either no oscillators, or involves at most states created from pure momenmtum carrying states by the action of $\psi^0_{-1/2}\bar\psi^0_{-1/2}$ is proportional to:

$$(f(X^0(0))+i\epsilon\psi^0_{-1/2}\bar\psi^0_{-1/2}g(X^0(0)))|0\rangle\,, \tag{4.6}$$

where f and g are the same functions defined in eqs. (3.7), (3.8). Thus the net level $(1/2,1/2)$ contribution to the boundary state is given by, upto a constant of proportionality,

$$i\left[\psi^0_{-1/2}\bar\psi^0_{-1/2}g(X^0(0))-\psi^j_{-1/2}\bar\psi^j_{-1/2}f(X^0(0))-(\bar\beta_{-1/2}\gamma_{-1/2}-\beta_{-1/2}\bar\gamma_{-1/2})f(X^0(0))\right]|\Omega\rangle\,. \tag{4.7}$$

Comparison of this with (3.3) immediately gives eq. (3.6).

Thus it remains to establish (4.6). We follow the approach of [1] to first study the boundary state in the Wick rotated theory $X^0\to iX$, $\psi^0\to i\psi$, $\bar\psi^0\to i\bar\psi$ and inverse Wick rotate after obtaining the boundary state. We shall not attempt to determine this state completely, but only determine the part that corresponds to a linear combination of the Fock vaccum states $|k\rangle$ carrying X momentum k, and states of the form $\psi_{-1/2}\bar\psi_{-1/2}|k\rangle$. These will be sufficient for establishing (4.6).

In the original theory we switch on a tachyon field proportional to $\cosh(x^0/\sqrt{2})$. In the Wick rotated theory this corresponds to a tachyon field proportional to $\cos(x/\sqrt{2})$. This gives rise to perturbation by a boundary term proportional to the integral of $\psi\sin(X/\sqrt{2})\otimes\sigma_1$ where σ_1 is an appropriate Chan-Paton factor [11]. We have $X_L=X_R$, $\psi=\bar\psi$ at the boundary. As in [11] we fermionize the field X through the relations:

$$X \equiv X_L+X_R\,, \tag{4.8}$$

$$e^{i\sqrt{2}X_R} = \frac{1}{\sqrt{2}}(\xi+i\eta)\,,\qquad e^{i\sqrt{2}X_L} = \frac{1}{\sqrt{2}}(\bar\xi+i\bar\eta)\,, \tag{4.9}$$

where ξ, η are right-moving Majorana fermions and $\bar\xi, \bar\eta$ are left moving Majorana fermions.[2] There is a natural SO(3) subgroup acting on ξ, η and ψ. We shall for convenience identify ξ, η and ψ axis as 1, 2 and 3 axis respectively. The boundary perturbation by $\psi\sin(X/\sqrt{2}) = \psi\sin(\sqrt{2}X_R)$ corresponds to perturbation by a term proportional to integral of $\psi\eta$.[3] The effect of this perturbation is an SO(3) rotation about the ξ axis. We shall represent it by an SU(2) group element:[4]

$$R = \begin{pmatrix} \cos(\pi\widetilde\lambda) & i\sin(\pi\widetilde\lambda) \\ i\sin(\pi\widetilde\lambda) & \cos(\pi\widetilde\lambda) \end{pmatrix} . \tag{4.10}$$

This corresponds to an SO(3) rotation angle of $2\pi\widetilde\lambda$.

In order to determine the NS-NS sector boundary state of the perturbed theory, we need to compute one point function of closed string vertex operators in this perturbed theory.[5] This can be computed by simply rotating the closed string vertex operator by the SU(2) rotation R, and computing the resulting one point function in the unperturbed theory. First we focus on the part involving linear combinations of Fock vacuum. Since the boundary perturbation is periodic in X with periodicity $2\pi\sqrt{2}$, we might expect that the boundary state will be a linear combination of states of momentum $n/\sqrt{2}$ with integer n. But the perturbation in fact has an additional symmetry $X \to 2\pi/\sqrt{2}$, $\psi \to -\psi$, $\bar\psi \to -\bar\psi$. Since the fermionic oscillators ψ_{-n}, $\bar\psi_{-n}$ always appear in pairs in the NS-NS sector boundary state, and hence is invariant under this transformation, we can conclude that the boundary state is in fact built on states carrying momentum $n\sqrt{2}$ for integer n. Thus we can write the oscillator free part of the boundary state as:

$$\sum_n \widetilde f_n \, |k = n\sqrt{2}\rangle \, , \tag{4.11}$$

where the sum over n runs over all integers, and $\widetilde f_n$ are coefficients to be determined. These are in fact proportional to the one point function of the closed string vertex operator $e^{-in\sqrt{2}X}$ on the disk. In order to carry out this computation we need to study how the SU(2) rotation by R affects this vertex operator. To determine this we note that as in [14, 15], the $e^{in\sqrt{2}X_R}$ part of the state $e^{in\sqrt{2}X} = e^{in\sqrt{2}(X_L+X_R)}$ transforms in the $(j = |n|, m = n)$ representation of the SU(2) group, whereas the part $e^{in\sqrt{2}X_L}$ remains unchanged. Under rotation by R this mixes with all the $(j = |n|, m)$ states in the representation, which can be represented as states created by combinations of ∂X_R and ψ oscillators acting on

[2] In writing (4.9) we have ignored the cocycle factors [12]. These could give rise to additional $\widetilde\lambda$ independent phases in the final answer. However, since we shall determine these phases by an independent argument anyway, we shall not include the cocycle factors in our analysis.

[3] We could also have chosen $\bar\psi\bar\eta$ since at the boundary $(\xi, \eta, \psi) = (\bar\xi, \bar\eta, \bar\psi)$.

[4] Note that the Chan-Paton factor σ_1 does not affect the NS-NS sector closed string vertex operator. In studying the effect of the deformation on one point function of closed string vertex operators, we could keep track of σ_1 by formally including it in the deformation parameter $\widetilde\lambda$. Since the final answer will be even in $\widetilde\lambda$, we can formally expand the answer in a power series in $\widetilde\lambda$ and then resum the series, and σ_1 drops out in this process.

[5] We could use the result of ref. [13] and reexpress the boundary state in terms of original coordinates, but we shall not do that here.

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$e^{i\sqrt{2}(nX_L+mX_R)}(0)|0\rangle$. However only the term proportional to $(j=|n|, m=-n)$ state, represented by the vertex operator $e^{in\sqrt{2}(X_L-X_R)}$, has a non-zero one point function, since in the unperturbed theory X has Neumann boundary condition. The coefficient of the state $(j=|n|, m=-n)$ under a rotation by R of $(j=|n|, m=n)$ is given by $D^{j=n}_{n,-n}(R)$, where $D^j_{m,m'}(R)$ is the spin j representation matrix of R in the J_z eigenbasis. Thus we have:

$$\widetilde{f}_n = D^{j=n}_{n,-n}(R)e^{i\varepsilon(n)}\,, \tag{4.12}$$

where $e^{i\varepsilon(n)}$ is a phase factor, appearing due to the fact that the choice of basis in which we compute $D^j_{m,m'}(R)$ may differ from the one we need by multiplicative phase factors, and also because we could get additional phases from various cocycle factors that we have ignored in fermionizing the boson X. We shall use the convention of [14] for $D^j_{m,m'}(R)$, which gives:

$$D^{j=|n|}_{n,-n}(R) = D^{j=|n|}_{-n,n}(R) = (i\sin(\widetilde{\lambda}\pi))^{2|n|} = (-1)^n\sin^{2|n|}(\widetilde{\lambda}\pi)\,. \tag{4.13}$$

Thus we get the component of $|\mathcal{B},\epsilon\rangle_{X,\psi}$ involving only momentum states to be proportional to

$$\left[1+\sum_{n=1}^{\infty}(-1)^n\sin^{2n}(\widetilde{\lambda}\pi)\left\{e^{i\varepsilon(n)}e^{i\sqrt{2}nX(0)}+e^{i\varepsilon(-n)}e^{-i\sqrt{2}nX(0)}\right\}\right]|0\rangle \tag{4.14}$$

where we have chosen $\varepsilon(0)=0$. According to [11], for $\widetilde{\lambda}=1/2$ this should reproduce the boundary state for an infinite array of D-branes with Dirichlet boundary condition along X, placed at the zeroes of the tachyon field $T(x)\propto\cos(x/\sqrt{2})$, i.e. at $x=\frac{2\pi}{\sqrt{2}}(n+\frac{1}{2})$.[6] It is easy to check that this happens provided we choose $\varepsilon(n)=0$.

After inverse Wick rotation $X\to -iX^0$, and setting $\varepsilon(n)=0$, we get the oscillator free part of the boundary state to be proportional to:

$$f(X^0(0))|0\rangle\,, \tag{4.15}$$

where

$$\begin{aligned} f(x^0) &= 1+\sum_{n=1}^{\infty}(-1)^n\sin^{2n}(\widetilde{\lambda}\pi)\left(e^{n\sqrt{2}x^0}+e^{-n\sqrt{2}x^0}\right) \\ &= \frac{1}{1+e^{\sqrt{2}x^0}\sin^2(\widetilde{\lambda}\pi)}+\frac{1}{1+e^{-\sqrt{2}x^0}\sin^2(\widetilde{\lambda}\pi)}-1\,. \end{aligned} \tag{4.16}$$

This gives eq. (3.7).

We now turn to the computation of $g(x^0)$. Instead of doing a full computation, we shall use a trick, and that is to use eq. (3.9) and energy conservation to conclude that $f(x^0)+g(x^0)$ must be conserved. Thus we must have:

$$g(x^0) = C - f(x^0)\,, \tag{4.17}$$

[6]Actually these are alternatively D-branes and anti-D-branes, but the NSNS sector boundary state is insensitive to this difference.

where C is an x^0 independent constant to be determined. We compute C by examining the power series expansion of g and f in powers of $e^{\pm\sqrt{2}x^0}$, and comparing the constant term on the two sides of eq. (4.17).

The constant term in f is 1, as seen from eq. (4.16). Equation (4.17) then tells us that the constant term in g is given by $C-1$. From eq. (4.6) we see that the coefficient of the $\psi^0_{-1/2}\bar\psi^0_{-1/2}|0\rangle$ term (relative to the coefficient of $|0\rangle$) in the expression for $|\mathcal{B},\epsilon\rangle_{X^0,\psi^0}$ is given by $i\epsilon(C-1)$. Thus after Wick rotation, the coefficient of $\psi_{-1/2}\bar\psi_{-1/2}|0\rangle$ in $|\mathcal{B},\epsilon\rangle_{X,\psi}$ will be $-i\epsilon(C-1)$. Hence to compute C, we need to compute the one point function of the closed string state $\psi_{-1/2}\bar\psi_{-1/2}|0\rangle$ on the disk in the perturbed theory and equate it to $-i\epsilon(C-1)$. In the unperturbed theory $|\mathcal{B},\epsilon\rangle_{X,\psi}$ is proportional to $\exp(-i\epsilon\psi_{-1/2}\bar\psi_{-1/2})|0\rangle$, and hence this one point function is given by $-i\epsilon$. Since the effect of the perturbation is an SO(3) rotation by angle $2\widetilde\lambda\pi$ on the (ψ,η) system without changing ξ, this converts $\psi_{-1/2}\bar\psi_{-1/2}|0\rangle$ to $(\cos(2\pi\widetilde\lambda)\psi_{-1/2}+\sin(2\pi\widetilde\lambda)\eta_{-1/2})\bar\psi_{-1/2}|0\rangle$. Thus we now need to compute the one point function of this in the unperturbed theory. Of this the one point function of $\eta_{-1/2}\bar\psi_{-1/2}|0\rangle$ vanishes. On the other hand, as already mentioned, the one point function of $\psi_{-1/2}\bar\psi_{-1/2}|0\rangle$ is $-i\epsilon$. Thus we get

$$-i\epsilon(C-1) = -i\epsilon\cos(2\pi\widetilde\lambda)\,. \tag{4.18}$$

This gives

$$C = 1+\cos(2\pi\widetilde\lambda)\,. \tag{4.19}$$

Using eqs. (4.17) and (4.19) we recover (3.8).

The tachyon perturbation proportional to $\widetilde\lambda\cosh(x^0/\sqrt{2})$ that we have analyzed here represents a system with total energy less than the tension of the unstable D-brane. If we want to consider systems with total energy larger than that of the unstable D-brane, we need to consider a tachyon configuration proportional to $\widetilde\lambda\sinh(x^0/\sqrt{2})$ [1]. This can be obtained from the previous background by the replacement $\widetilde\lambda\to -i\widetilde\lambda$, $x^0\to x^0+i\pi/\sqrt{2}$. Making these replacements in (4.16) we get

$$f(x^0) = \frac{1}{1+e^{\sqrt{2}x^0}\sinh^2(\widetilde\lambda\pi)} + \frac{1}{1+e^{-\sqrt{2}x^0}\sinh^2(\widetilde\lambda\pi)} - 1 \tag{4.20}$$

Thus $f(x^0)$ still vanishes as $x^0\to\infty$ and the system evolves to a pressureless gas.

Finally we note that we have focussed our attention only on the NS-NS sector of the boundary state since this is the part that carries information about the energy momentum tensor. Analysis of the RR sector of the boundary state may yield other important informations as in the analysis of [16].

5. Effective field theory analysis

In this section we shall give a description of the phenomenon observed in the earlier sections using an effective field theory.[7] The effective action for the tachyon field T on a D-p-brane

[7] I wish to thank B. Zwiebach for asking pertinent questions which led to this analysis.

that we shall be using is the one proposed in [17]:

$$S \propto -\int d^{p+1}x\, V(T)\sqrt{-\det A}\,, \tag{5.1}$$

where

$$A_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu T \partial_\nu T\,. \tag{5.2}$$

The tachyon potential $V(T)$ has a maximum at $T = 0$ and a minimum at $T = T_0$ where it vanishes. The relevant aspects of this model have been discussed in [18]. The energy momentum tensor $T_{\mu\nu}$ obtained from this action is:

$$T_{\mu\nu} \propto -V(T)\,\sqrt{-\det A}\,(A^{-1})_{\mu\nu}\,. \tag{5.3}$$

We now restrict to spatially homogeneous time dependent solutions for which $\partial_i T = 0$. Thus we have:

$$A_{00} = -1 + (\partial_0 T)^2\,, \qquad A_{ij} = \delta_{ij}\,, \qquad \det A = -1 + (\partial_0 T)^2\,, \tag{5.4}$$

$$(A^{-1})_{00} = (-1 + (\partial_0 T)^2)^{-1}, \qquad (A^{-1})_{ij} = \delta_{ij}\,, \tag{5.5}$$

and hence

$$T_{00} \propto V(T)\left(1 - (\partial_0 T)^2\right)^{-1/2}, \qquad T_{ij} \propto -V(T)\left(1 - (\partial_0 T)^2\right)^{1/2}\delta_{ij}, \qquad T_{i0} = 0\,. \tag{5.6}$$

Since T_{00} is conserved, we see from this that for a solution with fixed energy, as T approaches the minimum T_0 of the potential where $V(T_0) = 0$, $\partial_0 T$ must approach its critical value 1. This, in turn makes T_{ij} vanish in this limit. This is precisely what we have observed in the more rigorous boundary state analysis.

Note however that since $\partial_0 T$ approaches a finite constant 1, in this parametrization the minimum T_0 of the potential must be at infinity. Otherwise T will approach T_0 in a finite time.

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